

**Semester Two Examination, 2020**

**Question/Answer Booklet**

**MATHEMATICS**

**SOLUTIONS**

**METHODS**

**ATAR Year 12**

**Section One:**

**Calculator-free**

Student Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Please circle your teacher’s name

**Teacher: Miss Long Miss Rowden Ms Stone**

**Time allowed for this paper**

Reading time before commencing work: 5 minutes

Working time for paper: 50 minutes

**Materials required/recommended for this paper**

***To be provided by the supervisor***

Number of additional

answer booklets used

(if applicable):

This Question/Answer Booklet

Formula Sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Suggested working time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  |  |  |  | **Total** | 100 |

**Instructions to candidates**

1. The rules for the conduct of the ATAR course examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Supplementary pages for the use planning/continuing your answer to a question have been provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free 35% (52 Marks)**

This section has eight (8) questions. Answer **all** questions. Write your answers in the spaces

provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1 (7 marks)

(a) Determine an expression for when

(i) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ numerator   denominator |

(ii) . (3 marks)

|  |
| --- |
| **Solution** |
| *N.B. Simplifies to* |
| **Specific behaviours** |
|  derivative of   derivative of   correct expression using product rule |

(b) For the positive number , let .  
  
Determine the value(s) of for which . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expression for   correct value of |

Question 2 (7 marks)

The discrete random variable is defined by

(a) Determine the value of the constant . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ forms equation using and   correct value |

(b) Determine

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(ii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates   correct value |

(iii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates   correct value |

Question 3 (5 marks)

The rate of change of pressure in an air tank is given by , where is the

time in in minutes since it began to empty.

(a) Determine an expression for the pressure in the tank at any time . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correctly integrates   correct expression for |

(b) Determine

(i) the time taken for the pressure in the tank to fall to psi. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ simplifies equation to   correct time |

(ii) the minimum pressure in the tank for . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct pressure |

Question 4 (6 marks)

The continuous random variable takes values in the interval to and has **cumulative** distribution function where

(a) Determine

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(ii) the value of , if . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates   correct value |

(b) Determine , the probability density function of , and sketch the graph of .

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ (no penalty for just if sketch correct)   draws between endpoints   scaled and labelled axes |

Question 5 (7 marks)

The function is defined by .

The second derivative of is .

Determine the coordinates and nature of all stationary points of the graph of .

|  |
| --- |
| **Solution** |
| has a minimum at and a maximum at . |
| **Specific behaviours** |
| ✓ indicates correct use of quotient rule   correct   equates numerator to zero   determines -coordinates of stationary points   indicates correct use of second derivative for nature   correct minimum   correct maximum |

Question 6 (7 marks)

(a) Simplify . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expresses as single log   simplifies to number |

(b) Given that , determine the value of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ obtains multiple of   correct value |

(c) Determine the solution to the equation in the form . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes as log equation   factors out   solves and simplifies into required form |

Question 7 (6 marks)

The acceleration at time seconds of a small body travelling in a straight line is given by

When the body was at the origin and seconds later its displacement was cm.

Determine the velocity of the body when .

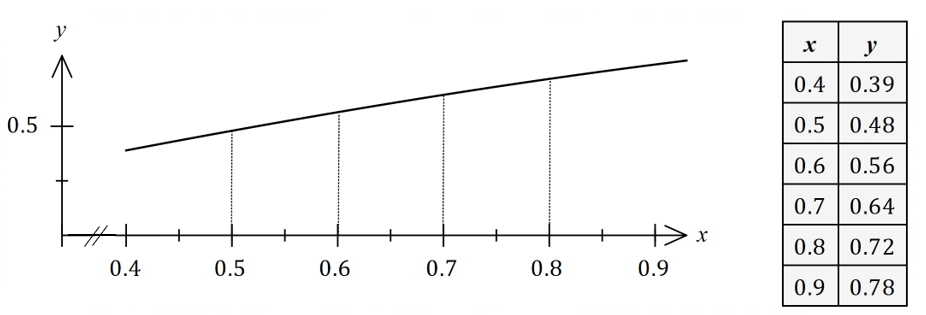
|  |
| --- |
| **Solution** |
| But |
| **Specific behaviours** |
| ✓ antiderivative of   integral for change in displacement   antiderivative of   simplifies equation for   uses given to determine value of   correct velocity |

Question 7 – alternative method

|  |
| --- |
| **Solution** |
| eqn |
| **Specific behaviours** |
| ✓ antiderivative of   antiderivative of   equation for  ✓ equation for   solves simultaneously to determine value of   correct velocity |

Question 8 (7 marks)

The graph and a table of values for is shown below, where .



Let .

(a) By using the information shown and considering sums of the form explain

why .

(3 marks)

|  |
| --- |
| **Solution** |
| With and then  Hence must be less than this value as it is the area of circumscribed rectangles that overestimate the area under the curve. |
| **Specific behaviours** |
| ✓ indicates -ordinates for circumscribed rectangles   shows sum of   explains inequality |

(b) In a similar manner to (a), determine the lower estimate,, for the value of .

That is, the value of for .

(2 marks)

|  |
| --- |
| **Solution** |
| With and then |
| **Specific behaviours** |
| ✓ indicates -ordinates for inscribed rectangles   value of |

(c) Use your previous answers to determine a numerical estimate for and explain whether your estimate is smaller or larger than the exact value of . (2 marks)

|  |
| --- |
| **Solution** |
| This value slightly underestimates the exact value of as the curve is concave downwards. |
| **Specific behaviours** |
| ✓ correctly averages values   states underestimate with reason |